Common Calculus Mistakes
Quotient Rule

Some problems provide the opportunity for more than one mistake.

The Goal

Find

$$\frac{d}{dz}\left(\frac{3z^2+1}{z^4-5}\right)$$

Mistake 1

Find the mistake:

$$\frac{d}{dz}\left(\frac{3z^2+1}{z^4-5}\right)=\frac{6z\cdot z^4-5\ -\ (3z^2+1)(4z^3)}{(z^4-5)^2}=\frac{-6z^5-4z^3-5}{(z^4-5)^2}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dz}\left(\frac{3z^2+1}{z^4-5}\right)=\frac{6z\cdot z^4-5\ -\ (3z^2+1)(4z^3)}{(z^4-5)^2}=\frac{-6z^5-4z^3-5}{(z^4-5)^2}$$

Correction 1

$$\frac{d}{dz}\left(\frac{3z^2+1}{z^4-5}\right)=\frac{6z(z^4-5)\ -\ (3z^2+1)(4z^3)}{(z^4-5)^2}=\frac{-6z^5-4z^3-30z}{(z^4-5)^2}$$

Mistake 2

Find the mistake:

$$\frac{d}{dz}\left(\frac{3z^2+1}{z^4-5}\right)=\frac{4z^3(3z^2+1)\ -\ (z^4-5)(6z)}{(z^4-5)^2}=\frac{6z^5+4z^3+30z}{(z^4-5)^2}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dz}\left(\frac{3z^2+1}{z^4-5}\right)=\frac{4z^3(3z^2+1)\ -\ (z^4-5)(6z)}{(z^4-5)^2}=\frac{6z^5+4z^3+30z}{(z^4-5)^2}$$

Correction 2

$$\frac{d}{dz}\left(\frac{3z^2+1}{z^4-5}\right) = \frac{6z(z^4-5) - (3z^2+1)(4z^3)}{(z^4-5)^2} = \frac{-6z^5-4z^3-30z}{(z^4-5)^2}$$

Explanations

In the first mistake parentheses should have been used around the factor z^4-5; not using them meant that the 6z factor was not distributed over the whole factor z^4-5. *Use parentheses!*

In the second mistake the numerator terms were reversed; this mistake changes the sign of the result. *Take care to get the order of the terms correct in the numerator when using the quotient rule.*